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PP-wave and non-supersymmetric gauge theory

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Abstract: We extend the pp-wave correspondence to a non supersymmetric example. The model is the type-0B string theory on the pp-wave R-R background. We explicitly solve the model and give the spectrum of physical states. The field theory counterpart is given by a sector of the large N $\mathrm{SU}(N) \times \mathrm{SU}(N)$ CFT living on a stack of N electric and N magnetic D3-branes. The relevant effective coupling constant is $g_{\mathrm{eff}} = g_s N/J^2$. The string theory has a tachyon in the spectrum, whose light-cone energy can be exactly computed as a function of g_{eff}. We argue that the perturbative analysis in g_{eff} in the dual gauge theory is reliable, with corrections of non perturbative type. We find a precise state/operator map, showing that the first perturbative corrections to the anomalous dimensions of the operators have the behavior expected from the string analysis.

Keywords: Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence.

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1. Introduction and conclusions

In [1] Berenstein, Maldacena and Nastase (BMN) studied the correspondence between type-IIB string theory and the large N, $\mathcal{N}=4$ SU(N) SCFT beyond the supergravity approximation. They found a precise map between *all the string modes* on a maximally supersymmetric plane wave background, and a sector of gauge invariant operators of the field theory. The string states are the ones with very large angular momentum ($J \sim \sqrt{N}$) along the equator of S^5 in the original $AdS_5 \times S^5$ background, which reduces to the pp-wave in the Penrose limit [2]. The resulting spacetime, which is supported by a null, constant five from field strength, has the form:

$$\begin{aligned} ds^2 &= 2dx^+dx^- - f^2x^2dx^+dx^+ + dx^Idx^I\,, \qquad I=1,\dots,8 \\ F_{+1234} &= F_{+5678} = 2f\,. \end{aligned} \tag{1.1}$$

The symmetry of the solution is SO(4) $\times$ SO(4).

Despite the presence of the R-R field, type-IIB string theory is exactly solvable on the pp-wave [3]. The spectrum, in light cone gauge, is obtained by acting on the vacuum with free oscillators. The light-cone hamiltonian is

$$H = p_+ = \sum_{n=-\infty}^{\infty} N_n \sqrt{f^2 + \frac{n^2}{(\alpha' p^+)^2}}\,, \tag{1.2}$$

where N_n is the occupation number of the n-th oscillator. This formula implies the following relation between the dimension Δ and the R-charge J of the corresponding gauge theory operator:

$$\Delta - J = \sum_{n=-\infty}^{\infty} N_n \sqrt{1 + \frac{4\pi g_s N n^2}{J^2}} \,. \tag{1.3}$$

This expression, valid for $\frac{\Delta - J}{J} \ll 1$, follows from the relations $p_+ = f(\Delta - J)$ and $p_- \approx \frac{J}{fR^2}$, where $R^4 \equiv 4\pi\alpha'^2 g_s N$ is the common radius of AdS_5 and S^5. In the Penrose limit R (and so $g_s N$), together with J and Δ, go to infinity, in such a way that p_+ and p_- (and so $\Delta - J$ and $\frac{J^2}{g_s N}$) stay finite.

BMN were able to identify the single-trace field theory operators with scaling dimensions given by (1.3). The operators corresponding to zero-mode states are chiral, so their scaling dimension is the same as in free field theory. The remaining operators are non-chiral, but their anomalous dimensions are predicted to be finite even in the strong coupling limit. The perturbative expansion for this class of almost BPS states is in fact controlled by $g_{\text{eff}} = \frac{\lambda}{J^2} = \frac{1}{4\pi(f\alpha' p^+)^2}$ rather than by the (large) 't Hooft coupling $\lambda = g_s N$. Anomalous dimensions can be perturbatively evaluated by expanding around the free field theory with effective coupling g_{eff}.

In this paper we analyze the pp-wave correspondence for a non supersymmetric model. The natural candidate to study is the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge theory introduced in [4]. It is dual to type-0B string theory on $AdS_5 \times S^5$, obtained as the near horizon geometry of a stack of N electric and N magnetic D3-branes. The gravity background supports a self-dual 5-form field strength, a constant dilaton and a constant (namely zero) tachyon field. We can thus perform the Penrose limit exactly as in the case of the type-IIB theory, ending up with type-0B on the pp-wave background (1.1). The theory consists of an untwisted sector, which is exactly the same as the bosonic part of the parent type-IIB string, and a twisted one, which has a tachyonic ground state. The light cone energy of the tachyon is a function of g_{eff}. It is a negative definite function, it diverges in the limit $g_{\text{eff}} \to \infty$, which closely mimics the flat space case, and it vanishes in the opposite free field theory limit $g_{\text{eff}} \to 0$.

This result deserves some comments. The original theory before the Penrose limit, type-0B on $AdS_5 \times S^5$, has a tachyonic instability at large λ associated with a complex dimension operator in the dual field theory [5]. However, this operator could be removed from the spectrum in the Penrose limit, as happens, for example, in [6]. The Penrose limit indeed selects only the operators with large, positive mass and scaling dimension $\Delta \sim \sqrt{\lambda}$. We can expect a similar behavior in our case, since the large and positive contribution of the angular momentum ($\sim J^2 = O(\lambda)$) should overwhelm the negative mass of the tachyonic ground state ($\sim -\sqrt{\lambda}$). Thus, all complex dimension field theory operators are not expected to survive the Penrose limit and $H_{AdS} \sim \Delta$ should be well-defined. The original instability of the AdS background is reflected in the fact that the spectrum of the pp-wave exhibits a light-cone energy $H_{\text{pp-wave}} \sim \Delta - J$ which can be infinitely negative in the gravity limit. It would be very interesting to understand if this unboundness of the light cone energy corresponds to a real instability in the string theory and to study its nature and origin directly in AdS and in the field theory.

One of the interesting aspects of the pp-wave background is that the energy of the tachyon as a function of the coupling $g_{\rm eff}$ can be exactly computed. As already mentioned, the light cone energy of the tachyon is always negative. More interestingly, the energy is zero for $g_{\rm eff} = 0$ and it is perturbatively zero to all orders in $g_{\rm eff}$. This means that the corresponding field theory is perturbatively stable around the free field theory limit. We conclude that we can study the correspondence between strings and operators in a *non-supersymmetric* theory order by order in a Feynman diagram expansion, as was done in the supersymmetric case in [1]. We can compare the behavior of the tachyon energy with the analogous situation in AdS. The mass of the tachyon in type-0B on AdS is a function of λ and, as was argued in [4], it should become positive for λ small enough, i.e. in these theories one can expect a transition from a stable to an unstable regime by varying the coupling constant. The transition is due to the effect of the RR five-form field, which contributes to the effective mass of the tachyon. In the pp-wave background there is no contribution from the null five-form field and the energy is always negative.

The paper is organized as follows. After a brief review of the type-IIB GS formalism on the pp-wave in Section two, in Section three we study the type- 0B string on the same background. The theory is obtained as an orbifold of type-IIB by $(-1)^F$, F being the spacetime fermion number. It has an untwisted and a twisted sector. In both of them we calculate the light-cone hamiltonian and we find that in the twisted sector the lowest state has negative energy. In Section four the spectrum of physical states is presented. In Section five we identify the gauge theory operators dual to the string states. We compare the predictions of the string hamiltonian with the planar corrections to the dimensions of the operators for small $g_{\rm eff}$, as suggested in [1], finding agreement both in the untwisted and twisted sector. We are not able to reproduce the part of the corrections to the classical dimension of the operators corresponding to the energy of the tachyon. It appears to involve a non-perturbative calculation in $g_{\rm eff}$. It would be interesting to investigate the twisted sector at nonzero string coupling [7, 8], in order to understand better the issue of stability in these type of non supersymmetric models. Finally, the identification of the ground state with a tachyon is reproduced by a gravity calculation in the appendix.

2. Type-IIB GS superstring on pp-wave

In this paper we follow the conventions of [3, 9]. The Green-Schwarz action for type-IIB superstring on the background (1.1) has a very simple form in light-cone gauge [3], where the following conditions (in units $2\pi\alpha' = 1$) are imposed:

$$\Gamma^+\theta^1 = \Gamma^+\theta^2 = 0\,, \qquad x^+ = p^+\tau\,. \tag{2.1}$$

Here $\theta^{1a}, \theta^{2a}, a = 1, \ldots, 16$ are two ten dimensional Majorana-Weyl spinors of the same chirality, and:

$$\Gamma^\pm = \frac{1}{\sqrt{2}}(\Gamma^9 \pm \Gamma^0)\,, \qquad (\Gamma^\pm)^2 = 0\,, \tag{2.2}$$

implying that only a half of the components of the fermionic fields are dynamical variables in light-cone gauge. Among the bosonic degrees of freedom, only the eight transverse ones,

$x^I, I = 1, \dots, 8$, are independent. The gauge-fixed equations of motion for the physical variables can be derived from the action:

$$S = \int d\tau d\sigma \left[\frac{1}{2} \left(\partial_+ x^I \partial_- x^I - \mathrm{m}^2 x_I^2 \right) + \mathrm{i} \left(\theta^1 \bar{\gamma}^- \partial_+ \theta^1 + \theta^2 \bar{\gamma}^- \partial_- \theta^2 - 2\mathrm{m} \theta^1 \bar{\gamma}^- \Pi \theta^2 \right) \right], \quad (2.3)$$

(where $\partial_\pm \equiv \partial_\tau \pm \partial_\sigma$, $\mathrm{m} \equiv p^+ \mathrm{f}$ and $\bar{\gamma}^+ \theta^{\mathcal{I}} = 0$)[1] supported by the constraint:

$$\int d\sigma \left[\partial_\tau x^I \partial_\sigma x^I + \mathrm{i} \left(\theta^1 \bar{\gamma}^- \partial_\sigma \theta^1 + \theta^2 \bar{\gamma}^- \partial_\sigma \theta^2 \right) \right] = 0, \quad (2.4)$$

which will enforce invariance under σ translations. The action (2.3) reduces to the light-cone GS one in flat spacetime when $\mathrm{f} = 0$. When $f \neq 0$ it simply describes eight free massive 2d scalar fields x^I and eight free massive 2d Majorana fermionic fields (with real components θ^1, θ^2) whose equations of motion read:

$$\partial_+ \partial_- x^I + \mathrm{m}^2 x^I = 0, \qquad \partial_+ \theta^1 - \mathrm{m} \Pi \theta^2 = 0, \qquad \partial_- \theta^2 + \mathrm{m} \Pi \theta^1 = 0. \quad (2.5)$$

With the use of the fermionic equations of motion, the light-cone hamiltonian can be cast in the form [3]:

$$H = \frac{1}{p^+} \int_0^1 d\sigma \left[\frac{1}{2} \left(\pi_I^2 + (\partial_\sigma x_I)^2 + \mathrm{m}^2 x_I^2 \right) + \mathrm{i} (\theta^1 \bar{\gamma}^- \partial_\tau \theta^1 + \theta^2 \bar{\gamma}^- \partial_\tau \theta^2) \right], \quad (2.6)$$

where $\pi_I = \partial_\tau x_I$ are the bosonic canonical momenta.

3. Type-0B model and its quantization

In order to obtain the type-0B model we perform a quotient of the type-IIB theory by $(-1)^F$, where F is the spacetime fermion number. In the GS formulation this corresponds to the projection:

$$\theta \approx -\theta \quad (3.1)$$

on the fermionic coordinates. Just as in ordinary orbifolds, this gives us two sectors.

In the *untwisted sector* worldsheet bosons and fermions obey periodic boundary conditions:

$$x^I(\sigma + 1, \tau) = x^I(\sigma, \tau), \qquad \theta(\sigma + 1, \tau) = \theta(\sigma, \tau), \qquad 0 \le \sigma \le 1. \quad (3.2)$$

From these one can get [9] the following general solution of the equations of motion (2.5):

$$x^I(\sigma, \tau) = \cos \mathrm{m}\tau \, x_0^I + \mathrm{m}^{-1} \sin \mathrm{m}\tau \, p_0^I + \mathrm{i} \sum_{n \neq 0} \frac{1}{\omega_n} \left(\varphi_n^1(\sigma, \tau) \alpha_n^{1I} + \varphi_n^2(\sigma, \tau) \alpha_n^{2I} \right), \quad (3.3)$$

$$\theta^1(\sigma, \tau) = \cos \mathrm{m}\tau \, \theta_0^1 + \sin \mathrm{m}\tau \, \Pi \theta_0^2 + \sum_{n \neq 0} c_n \left(\varphi_n^1(\sigma, \tau) \theta_n^1 + \mathrm{i} \tfrac{\omega_n - k_n}{\mathrm{m}} \varphi_n^2(\sigma, \tau) \Pi \theta_n^2 \right), \quad (3.4)$$

$$\theta^2(\sigma, \tau) = \cos \mathrm{m}\tau \, \theta_0^2 - \sin \mathrm{m}\tau \, \Pi \theta_0^1 + \sum_{n \neq 0} c_n \left(\varphi_n^2(\sigma, \tau) \theta_n^2 - \mathrm{i} \tfrac{\omega_n - k_n}{\mathrm{m}} \varphi_n^1(\sigma, \tau) \Pi \theta_n^1 \right), \quad (3.5)$$

[1]Here, just as in [3, 9], γ^m, $\bar{\gamma}^m$ are the 16×16 Dirac matrices which are the off-diagonal parts of 32×32 matrices Γ^m. The matrix Π ($\Pi^2 = 1$) is the product of four γ-matrices and have its origin from the coupling with the five form background field strength.

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where:

$$\varphi_n^1(\sigma,\tau) = \exp\left(-\mathrm{i}(\omega_n\tau - k_n\sigma)\right), \qquad \varphi_n^2(\sigma,\tau) = \exp(-\mathrm{i}(\omega_n\tau + k_n\sigma)) \tag{3.6}$$

and

$$\omega_n = \sqrt{k_n^2 + \mathrm{m}^2}\,, \qquad n > 0\,; \qquad \omega_n = -\sqrt{k_n^2 + \mathrm{m}^2}\,, \qquad n < 0\,; \tag{3.7}$$

$$k_n \equiv 2\pi n\,, \qquad c_n = \frac{1}{\sqrt{1 + (\frac{\omega_n - k_n}{\mathrm{m}})^2}}\,, \qquad n = \pm 1, \pm 2, \dots\,. \tag{3.8}$$

In the *twisted sector* the worldsheet fermions get antiperiodic boundary conditions:

$$\theta(\sigma+1,\tau) = -\theta(\sigma,\tau)\,, \qquad 0 \le \sigma \le 1\,. \tag{3.9}$$

Thus the corresponding solutions of the equations of motion are (those for the bosons are exactly the same as in the untwisted sector):

$$\theta^1(\sigma,\tau) = \sum_r c_r\left(\varphi_r^1(\sigma,\tau)\theta_r^1 + \mathrm{i}\tfrac{\omega_r - k_r}{\mathrm{m}}\varphi_r^2(\sigma,\tau)\Pi\theta_r^2\right), \tag{3.10}$$

$$\theta^2(\sigma,\tau) = \sum_r c_r\left(\varphi_r^2(\sigma,\tau)\theta_r^2 - \mathrm{i}\tfrac{\omega_r - k_r}{\mathrm{m}}\varphi_r^1(\sigma,\tau)\Pi\theta_r^1\right). \tag{3.11}$$

Here the index r runs over the *half-integer* numbers, and, due to the antiperiodic boundary condition, no zero mode appears.

The quantization of this model is straightforward. The physical states are required to be invariant under $\theta \to -\theta$ and to have zero-eigenvalue under the operator (2.4).

The hamiltonian (2.6) aquires a very simple form in terms of the following operators (see [9]):

$$a_0^I = \frac{1}{\sqrt{2\mathrm{m}}}(p_0^I + \mathrm{im}x_0^I)\,, \qquad \bar{a}_0^I = \frac{1}{\sqrt{2\mathrm{m}}}(p_0^I - \mathrm{im}x_0^I)\,, \tag{3.12}$$

$$\alpha_{-n}^I = \sqrt{\frac{\omega_n}{2}}\,a_n^I\,, \qquad \alpha_n^I = \sqrt{\frac{\omega_n}{2}}\,\bar{a}_n^I\,, \qquad n = 1, 2, \dots \tag{3.13}$$

$$\theta_R = \frac{1+\Pi}{2}(\theta_0^1 + i\theta_0^2)\,, \qquad \theta_L = \frac{1-\Pi}{2}(\theta_0^1 + i\theta_0^2)\,, \tag{3.14}$$

$$\theta_{-q}^{\mathcal{I}} \equiv \frac{1}{\sqrt{2}}\eta_q^{\mathcal{I}}\,, \qquad \theta_q^{\mathcal{I}} \equiv \frac{1}{\sqrt{2}}\bar{\eta}_q^{\mathcal{I}}\,, \tag{3.15}$$

(here q is integer and not zero, or half-integer) whose (anti)commutation algebra reads:

$$[\bar{a}_0^I, a_0^J] = \delta^{IJ}\,, \qquad [\bar{a}_m^{\mathcal{I}I}, a_n^{\mathcal{J}J}] = \delta_{m,n}\delta^{IJ}\delta^{\mathcal{I}\mathcal{J}}\,, \tag{3.16}$$

$$\{\bar{\eta}_q^{\mathcal{I}a}, \eta_p^{\mathcal{J}b}\} = \frac{1}{2}(\gamma^+)^{ab}\delta_{q,p}\delta^{\mathcal{I}\mathcal{J}}\,, \qquad \{\bar{\theta}_R, \theta_L\} = 0\,, \tag{3.17}$$

$$\{\bar{\theta}_R, \theta_R\} = \frac{(1+\Pi)}{4}\gamma^+\,, \qquad \{\bar{\theta}_L, \theta_L\} = \frac{(1-\Pi)}{4}\gamma^+\,. \tag{3.18}$$

Correspondingly we define the Fock vacuum in each sector as:

$$\bar{a}_0^I|0,p^+\rangle_{U,T} = 0\,, \qquad \bar{a}_n^{\mathcal{I}I}|0,p^+\rangle_{U,T} = 0\,, \qquad \bar{\eta}_q^{\mathcal{I}\alpha}|0,p^+\rangle_{U,T} = 0\,, \tag{3.19}$$

$$\theta_R|0,p^+\rangle_U = 0\,, \qquad \bar{\theta}_L|0,p^+\rangle_U = 0\,. \tag{3.20}$$

As it was outlined in [9], the above choice for the fermionic zero-mode vacuum is the one which reflects the $\mathrm{SO}(4) \times \mathrm{SO}(4)$ symmetry of the pp-wave background.

The hamiltonian in the untwisted sector. In the above basis the light-cone hamiltonian in the untwisted sector reads (we restore here the α' dependence):

$$H_U = H_{0,U} + \frac{1}{\alpha' p^+} \sum_{\mathcal{I}=1,2} \sum_{n=1}^{\infty} \sqrt{n^2 + (\alpha' p^+ \mathrm{f})^2} \; (a_n^{\mathcal{I}I} \bar{a}_n^{\mathcal{I}I} + \eta_n^{\mathcal{I}} \bar{\gamma}^- \bar{\eta}_n^{\mathcal{I}}) \,. \tag{3.21}$$

The untwisted zero-mode hamiltonian $H_{0,U}$ gets contributions from the normal ordering of fermionic and bosonic zero-modes. The normal ordering of the non-zero modes cancels between bosons and fermions, which appear in equal number. The general expression of $H_{0,U}$ is:

$$H_{0,U} = \mathrm{f} \left(a_0^I \bar{a}_0^I + \theta_L \bar{\gamma}^- \bar{\theta}_L - \theta_R \bar{\gamma}^- \bar{\theta}_R + e_0 \right) . \tag{3.22}$$

The choice (3.20) for the vacuum amounts to setting $e_0 = 0$ [9].

In summary, the hamiltonian in the type-0B untwisted sector is exactly the same as in the type-IIB case. Differences will appear in the space of physical states.

The hamiltonian in the twisted sector. In the twisted sector we get the following expression for the light-cone hamiltonian:

$$H_T = H_{0,T} + \frac{1}{\alpha' p^+} \sum_{\mathcal{I}=1,2} \left[\sum_{n=1}^{\infty} \sqrt{n^2 + (\alpha' p^+ \mathrm{f})^2} \; a_n^{\mathcal{I}I} \bar{a}_n^{\mathcal{I}I} + \sum_{r=1/2}^{\infty} \sqrt{r^2 + (\alpha' p^+ \mathrm{f})^2} \; \eta_r^{\mathcal{I}} \bar{\gamma}^- \bar{\eta}_r^{\mathcal{I}} \right] . \tag{3.23}$$

The zero-point hamiltonian $H_{0,T}$ now gets a non-trivial contribution from the normal ordering of the fermionic and bosonic non-zero modes, bacause the fermions here obey antiperiodic boundary conditions. Explicitely we have $H_{0,T} = \mathrm{f}(a_0^I \bar{a}_0^I) + E_{0,T}$, with:

$$E_{0,T} = 4\mathrm{f} + \frac{8}{\alpha' p^+} \left[\sum_{n=1}^{\infty} \sqrt{n^2 + (\alpha' p^+ \mathrm{f})^2} - \sum_{r=1/2}^{\infty} \sqrt{r^2 + (\alpha' p^+ \mathrm{f})^2} \right] , \tag{3.24}$$

where the first addendum is the normal ordering constant of the 8 bosonic zero modes.

In order to evaluate the difference of the diverging series in the previous expression, we make use of the *Epstein function* (the explicit representation we consider here is one of the possible and is taken from [10]):

$$\begin{aligned} F[z, s, m^2] &= \sum_{n=1}^{\infty} \left[(n+s)^2 + m^2 \right]^{-z} = \\ &= \frac{1}{2} [(s+1)^2 + m^2]^{-z} + \int_1^{\infty} dx [(x+s)^2 + m^2]^{-z} + \\ &\quad + i \int_0^{\infty} dt \left[\frac{[(1+it+s)^2 + m^2]^{-z} - [(1-it+s)^2 + m^2]^{-z}}{e^{2\pi t} - 1} \right] . \end{aligned} \tag{3.25}$$

This has a simple pole when $z = 1/2, -1/2, -3/2, \ldots$. We are interested in $z = -1/2$. The pole cancels in $E_{0,T}$ due to the opposite contribution of bosons and fermions. The result is (let us note that $s = 0$ for the bosonic series, and $s = -1/2$ in the fermionic case):

$$E_{0,T} = 4\mathrm{f} - \frac{1}{\alpha' p^+}\left[\sqrt{1+4(f\alpha' p^+)^2} + \right.$$
$$\left. + 4(f\alpha' p^+)^2 \log\left[\frac{2+2\sqrt{1+(f\alpha' p^+)^2}}{1+\sqrt{1+4(f\alpha' p^+)^2}}\right] - 8I(f\alpha' p^+)\right], \qquad (3.26)$$

where:

$$I(f\alpha' p^+) = i\int_0^\infty dt\left[\frac{f(0;1+it)-f(0;1-it)-f(-1/2;1+it)+f(-1/2;1-it)}{e^{2\pi t}-1}\right], \qquad (3.27)$$

and:

$$f(s;x) = \sqrt{(x+s)^2+(f\alpha' p^+)^2}\,. \qquad (3.28)$$

In the flat space limit $f = 0$, we recover the standard type-0B zero-point light-cone hamiltonian:

$$H_{0,T}(f=0) = -\frac{1}{\alpha' p^+}\,, \qquad (3.29)$$

which signals the presence of a tachyonic field of mass squared $m^2 = -2/\alpha'$ corresponding to the Fock vacuum of the twisted sector.

For $f \neq 0$, $E_{0,T}$ is a non-analytic function of $g_{\rm eff}^{-1/2} \sim (\alpha' p^+ f)$. For $g_{\rm eff} = 0$ it is zero, and this value does not receive any ("perturbative") correction in powers of $g_{\rm eff}$: order by order the energy is zero. The twisted zero-mode energy is thus non-perturbative in $g_{\rm eff}$.[2] $E_{0,T}$ is always negative, approaching zero for $g_{\rm eff} \to 0$ and $4f - \frac{1}{\alpha' p^+}$ for $g_{\rm eff} \to \infty$.

4. The spectrum of physical states

Generic Fock space vectors in both sectors are built up in terms of products of creation operators acting on the vacuum. The subspace of physical states is obtained by imposing the constraint (2.4), which in terms of the oscillators reads [9]:

$$N^1|\Phi_{phys}\rangle = N^2|\Phi_{phys}\rangle\,, \qquad N^{\mathcal{I}} = \sum_{n=1}^{\infty} k_n a_n^{\mathcal{I}I}\bar{a}_n^{\mathcal{I}I} + \sum_q k_q \eta_q^{\mathcal{I}}\bar{\gamma}^- \bar{\eta}_q^{\mathcal{I}}\,. \qquad (4.1)$$

In the type-0B model, however, this is not the only constraint to impose. In fact, the spectrum of physical states is obtained by projecting out all the states which are not invariant under the orbifold projection $\theta \to -\theta$. So, just as in the flat case, *we will not have spacetime fermions in the spectrum*: all the states obtained from the vacuum by acting with an odd number of fermionic operators are projected out.

In the *untwisted* sector the generic state built only in terms of fermionic zero modes is:

$$(\bar{\theta}_R)^{n_R}(\theta_L)^{n_L}|0,p^+\rangle_U\,, \qquad n_L, n_R = 0,1,2,3,4\,, \qquad (4.2)$$

[2] The nature of the "non perturbative" corrections is clearer in the form of the analytic continuation of $F[z,s,m^2]$ that can be found, for example, in [11]. In that paper the corrections take the form of Bessel functions, with an exponential vanishing behavior.

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where the limited range of values of n_R, n_L is due to the relations $(\bar\theta_R)^5 = (\theta_L)^5 = 0$. Also, the type-0 projection requires $n_L + n_R$ to be even. Acting with the hamiltonian operator on the states (4.2) we find:

$$H_U(\bar\theta_R)^{n_R}(\theta_L)^{n_L}|0,p^+\rangle_U = \mathrm{f}(n_R+n_L)(\bar\theta_R)^{n_R}(\theta_L)^{n_L}|0,p^+\rangle_U\,. \tag{4.3}$$

Thus the lowest energy state is just the Fock vacuum which has zero light-cone energy. This state corresponds to a purely gravity mode which is obtained from a linear combination of the trace of the graviton and the four-form potential [9]. The other states, with energies $2f, 4f, 6f, 8f$ (which are all zero in the flat limit), complete the untwisted gravity sector. The action of the bosonic zero modes gives for each state a tower of states with higher energies. Unlike the zero-mode sector, all the higher oscillators produce states whose light-cone energy is a function of $g_{\rm eff}^{-1/2} \sim (f\alpha' p^+)$. Their energy becomes infinite in the gravity limit $g_{\rm eff} \to \infty$, and they decouple from the gravity modes, as appropriate for stringy modes in a weakly curved background. On the other hand, zero modes and excited oscillator states are almost degenerate in the highly curved background limit $g_{\rm eff} \to 0$. This is the regime where comparison with a perturbative gauge theory can be done.

The spectrum of physical states is just the bosonic part of the corresponding type-IIB spectrum.

In the *twisted sector*, as we have anticipated, the Fock vacuum energy $E_{0,T}$ (3.26) depends non-trivially on $g_{\rm eff}$ and is always negative. In the gravity limit $g_{\rm eff} \to \infty$

$$E_{0,T}|0,p^+\rangle_T \approx \left(4f - \frac{1}{\alpha' p^+}\right)|0,p^+\rangle_T \tag{4.4}$$

and we can interpret the two terms in the previous formula as the effect of the curved spacetime $(4f)$ and the contribution of the "mass" squared of a tachyonic scalar field $(-2/\alpha')$. In the appendix formula (4.4) is derived in the gravity approximation. In type-0B, the negative mass squared of the tachyon in the gravity limit $g_{\rm eff} \to 0$ is so large to overwhelm the positive contribution of the curvature, and we are left with an infinitely negative twisted ground state energy. When $f = 0$ this state corresponds to the usual type-0B tachyon. As in the untwisted sector, the action of the bosonic zero modes gives a tower for each state in the spectrum.

The first excited states are obtained by acting on the vacuum with $\eta^1_{1/2}\eta^2_{1/2}$. The corresponding value of the light-cone hamiltonian depends non-trivially on $g_{\rm eff}$, but now it is always positive. In the gravity limit $g_{\rm eff} \to \infty$ their energy goes to the finite value $4f$. When $f = 0$ this is zero and these states correspond to massless fields of the twisted R-R sector. Note that the untwisted sector R-R fields have energies that are independent of $g_{\rm eff}$ and are different from each other. On the other hand, the energies of the twisted R-R forms are equal and vary with $g_{\rm eff}$. The two sets of R-R fields appear to be quite different, so it would be interesting to study if there exist combinations coupling to electric and magnetic branes as in the flat space case. All other excited string states decouple in the gravity limit.

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In the opposite limit $g_{\text{eff}} \to 0$ the energy of the tachyon is zero, and the energy of the R-R fields is $2f$. Just as in the untwisted sector, all string states are almost degenerate. In this regime, we can expect to be able to compare the string theory with the field theory results.

5. Field theory interpretation

The dual to the type-0B String on $AdS_5 \times S^5$ with self dual R-R five form flux is conjectured to be a large N conformal non supersymmetric gauge theory with group $\mathrm{SU}(N) \times \mathrm{SU}(N)$ that, apart from the gauge bosons, consists of six scalars ϕ^i, $\tilde{\phi}^i$, $i = 1,\ldots,6$, (the tilde distinguishes the two gauge groups) in the adjoint representation of each of the two groups (that have the same coupling constant), four bifundamental Weyl spinors χ^α in the $(\mathrm{N}, \bar{\mathrm{N}})$ and four, ψ^α, in the $(\bar{\mathrm{N}}, \mathrm{N})$ [4]. This theory can be obtained from the $\mathcal{N} = \triangle\,\mathrm{SU}(\in\mathcal{N})$ SYM by a Z_2 projection, just as type-0B can be obtained from an orbifold of type-IIB. If we arrange the $\mathcal{N} = \triangle$ fields in $2\mathrm{N} \times 2\mathrm{N}$ matrices, the projection is made with $(-1)^F$, where F is the fermion number, together with a conjugation by $\mathcal{I} = \mathrm{diag}(\mathcal{I}, -\mathcal{I})$, I being the $\mathrm{N} \times \mathrm{N}$ identity matrix [12]. As was shown in [12] this construction implies that the orbifold group lives in the center of SU(4). This leaves the diagonal $\mathrm{N} \times \mathrm{N}$ blocks for the bosons and the off-diagonal ones for the fermions. The operators of the theory can be organized in untwisted and twisted ones, depending on the behavior under the projection: the formers are even under exchange of the two gauge groups, the latter are odd. They can be written as traces of products of the $2\mathrm{N} \times 2\mathrm{N}$ matrices representing the $\mathcal{N} = 4$ fields, inserting $\mathcal{I}$ when dealing with twisted operators. For example, the $\mathcal{N} = 4$ operator $\chi Z \chi Z$, gives rise in the twisted sector to operators of the form:

$$\mathrm{tr}\left[\begin{pmatrix} 0 & \chi \\ \psi & 0 \end{pmatrix}\begin{pmatrix} Z & 0 \\ 0 & \tilde{Z} \end{pmatrix}\begin{pmatrix} 0 & \chi \\ \psi & 0 \end{pmatrix}\begin{pmatrix} Z & 0 \\ 0 & \tilde{Z} \end{pmatrix}\mathcal{I}\right] = \chi\tilde{Z}\psi Z - \psi Z \chi \tilde{Z}\,, \tag{5.1}$$

where we omitted the trace over the gauge indexes. Our notations follow [1], so, for example, $Z = \frac{1}{\sqrt{2}}(\phi^5 + i\phi^6)$.

The untwisted sector inherits the non-renormalization properties of the parent $\mathcal{N} = \triangle$ theory. In fact, the correlators of the untwisted sector operators in the planar limit are exactly the same as the corresponding correlators in $\mathcal{N} = \triangle$ SYM [13, 12]. The identification of operators with string states in the untwisted sector therefore is the same as that for the bosonic part of the type-IIB model.

The twisted sector is not protected by any $\mathcal{N} = \triangle$ heritage. For large 't Hooft coupling constant λ, the AdS spectrum contains an operator with complex dimension, making the whole theory unphysical [5]. This operator is precisely the one whose source is the bulk tachyon. It has also been argued in [14] that the theory is unstable also for small λ due to a scalar potential destabilizing the vacuum. The pp-wave limit corresponds to large λ *and* J values. In this regime, as anticipated in the Introduction, we don't expect any operator with complex dimension. The reason is that only high harmonics (with charge J) of the tachyon survive the limit. These highly excited states have an effective mass squared (measured in units of the AdS radius) of order $-2\sqrt{\lambda}+16+J(J+4)$, where the first term is the flat space value for the tachyon mass, the second one comes from the coupling with the AdS five form[3] and the third one comes from the angular momentum on the five-sphere. Recalling that $J \sim \sqrt{\lambda}$, the masses are expected to satisfy the Breitenlohner-Freedman bound on AdS. If we (naively) extrapolate the mass/dimension formula of the AdS/CFT dictionary to the pp-wave regime, we are left with positive dimension operators. In fact we do not encounter any instability in Δ, which is positive and of order $\sqrt{\lambda}$. We find instead that $\Delta - J$ is unbounded from below. This phenomenon presumably descends from the instabilities of the original theory, but we have no reliable analysis at hand.

5.1 Untwisted sector

The string ground state has $H_U = 0$. The only operator in the untwisted sector with $\Delta - J = 0$ is $\mathrm{Tr}[Z^J] + \mathrm{Tr}[\tilde{Z}^J]$. In the planar limit its dimension is expected to be protected as for the corresponding BPS $\mathcal{N} = \triangle$ operator. We identify it with the string ground state:

$$|0, p^+\rangle_U \longleftrightarrow \frac{1}{\sqrt{2JN^J}}(\mathrm{Tr}[Z^J] + \mathrm{Tr}[\tilde{Z}^J])\,. \tag{5.2}$$

The identification of the states constructed acting on the ground state with the zero modes and with the oscillators is the same as that in the type-IIB theory. The action of the zero modes on the vacuum corresponds to operators whose dimension is also protected. As a general rule, in both the untwisted and twisted sectors, the action of the bosonic oscillators $a^{\mathcal{I}I}$ for $I = 5, ldots\,, 8$ corresponds to insertions in the correlators of the fields ϕ^{I-4}, while for $I = 1, \ldots, 4$ to insertions of $D_I Z$. The insertions of fields with $\Delta - J > 1$ give rise to operators that decouple in the pp-wave limit. As an example of zero mode insertion we have:

$$a_0^7|0, p^+\rangle_U \longleftrightarrow \frac{1}{\sqrt{2N^{J+1}}}(\mathrm{Tr}[\phi^3 Z^J] + \mathrm{Tr}[\tilde{\phi^3}\tilde{Z}^J])\,. \tag{5.3}$$

In the same way, as an example of the first excited string states, one has:

$$a_n^{1,8} a_n^{2,7}|0, p^+\rangle_U \longleftrightarrow \frac{1}{\sqrt{2JN^{J+2}}}\left(\sum_{l=0}^{J} \mathrm{Tr}\left[\phi^4 Z^l \phi^3 Z^{J-l}\right] e^{\frac{2\pi i n l}{J}} + \sum_{l=0}^{J} \mathrm{Tr}\left[\tilde{\phi}^4 \tilde{Z}^l \tilde{\phi}^3 \tilde{Z}^{J-l}\right] e^{\frac{2\pi i n l}{J}}\right). \tag{5.4}$$

As in the $\mathcal{N} = \triangle$ case, the classical dimension of the stringy states is perturbatively corrected. The perturbative series in g_{eff} is re-summed by the square root in (3.21).

The action of the fermionic oscillators on the vacuum, corresponding to the insertion of the bifundamental spinors, involves combinations of factors of the form $\mathrm{Tr}[\chi \tilde{Z}^l \psi Z^{J-l}]$ (the projection forces always the presence of an even number of spinors, giving only bosonic operators). For example, the vacuum state acted on by two fermionic zero-modes corresponds to the phase-unweighted operator:[4]

$$\bar{\theta}_R^a \theta_L^b|0, p^+\rangle_U \longleftrightarrow \frac{1}{\sqrt{2JN^{J+2}}}\left(\sum_{l=0}^{J} \mathrm{Tr}\left[\chi^a \tilde{Z}^l \psi^b Z^{J-l}\right] + \sum_{l=0}^{J} \mathrm{Tr}\left[\psi^a Z^l \chi^b \tilde{Z}^{J-l}\right]\right), \tag{5.5}$$

[3] Notice that in the pp-wave background the five form does not give any contribution to the mass of the tachyon.

[4] It has protected dimension. This is not the case for the "minus-sign" counterpart, which will receive perturbative corrections in λ and will decouple in the pp-wave limit.

where $a,b=1,\dots,8$, on the operator side count only the $J=\frac{1}{2}$ components of the spinors.

The excited string states correspond to:

$$\eta_n^{1,a}\eta_n^{2,b}|0,p^+\rangle_U \longleftrightarrow \frac{1}{\sqrt{2JN^{J+2}}}\left(\sum_{l=0}^{J}\mathrm{Tr}\left[\chi^a\tilde{Z}^l\psi^b Z^{J-l}\right]e^{\frac{2\pi inl}{J}}+\sum_{l=0}^{J}\mathrm{Tr}\left[\psi^a Z^l\chi^b\tilde{Z}^{J-l}\right]e^{\frac{2\pi inl}{J}}\right). \tag{5.6}$$

The first correction in g_{eff} to the dimension of the operators (5.4) and (5.6) has the correct behavior to reproduce the first term in the expansion of the square root in (3.21). All relevant contributions to the anomalous dimensions come from contracting each term in (5.4) and (5.6) with itself. Possible off-diagonal contributions — for example, the contraction of the l terms in the first sum in (5.6) with the $J-l$ term in the second sum — are suppressed by a $1/J$ factor, due to the phase factors. The calculation is then identical to the one in [1]. Graphs with fermions contains two Yukawa interactions.

5.2 Twisted sector

The energy of the string ground state, given by formula (3.26), is a function of $(f\alpha' p^+)^2 \sim \frac{1}{g_{\mathrm{eff}}}$, as we have seen. For $g_{\mathrm{eff}}=0$ it is zero. The function $E_{0,T}$ is not analytic in g_{eff}, and there are no power corrections in g_{eff} around $g_{\mathrm{eff}}=0$. This means that the dual operator has no perturbative corrections (in g_{eff}) to its classical dimension. $\Delta-J$ is therefore zero. The operator is:

$$|0,p^+\rangle_T \longleftrightarrow \frac{1}{\sqrt{2JN^J}}(\mathrm{Tr}[Z^J]-\mathrm{Tr}[\tilde{Z}^J])\,. \tag{5.7}$$

Note that in AdS the operator dual to the tachyon is identified with $\mathrm{Tr}(F_1^2)-\mathrm{Tr}(F_2^2)+\cdots$. In a sense, the AdS tachyon is the twisted sector counterpart of the operator dual to the dilaton $e^\phi \to \mathrm{Tr}(F_1^2)+\mathrm{Tr}(F_2^2)+\cdots$. In the pp-wave background, instead, the operator dual to the tachyon contains only the Z scalars[5]. This is consistent with the fact that the zero-mode of the AdS tachyon couples to F^2, while the higher harmonics couple to scalar fields [15, 14]. This is also manifest in the form of the Born-Infeld lagrangian in Einstein frame:

$$\int d^4x V(T)\sqrt{|g+e^{-\phi/2}F|} \tag{5.8}$$

when the tachyon field is Taylor expanded, $T=\sum T_n\phi_{\{i_1}\dots\phi_{i_n\}}$.

Reconstructing the function (3.26) in field theory is not an easy task. First of all, as we have seen, we need to consider non perturbative g_{eff} corrections. Moreover we should work in the strongly coupled λ regime. Note that, being (3.26) non analytic, we don't expect to be able to extrapolate the perturbative λ calculations to the strong regime; of course, we can't rely on supersymmetry to protect our results. As a check of the validity of the identification, we note that to loop order J the operator has no perturbative corrections.

[5]The pp-wave dilaton is identified with the untwisted ground state acted upon by four fermionic zero modes [9]. In AdS a dilaton harmonic with charge J is dual to an operator containing, among others, terms of the form $\mathrm{Tr}(F^2Z^{J-2}+\chi^4Z^{J-4}+\dots)$. In the large J limit, the leading contributions to correlation functions come from the terms with arbitrary insertion of four fermions in the string of $Z's$, consistently with the string picture.

Any difference with the untwisted operator (5.2), which is protected, resides in interactions of the form $\mathrm{Tr}[Z^J]\,\mathrm{Tr}[\tilde{Z}^J]$, that give the first contributions at order λ^J. Contribution of this order are problematic in the BMN approach and are usually neglected [8]. We interpret our result as an indication that these λ^J corrections disappear in the large 't Hooft coupling regime.

Operators in the twisted sector contain an extra minus-sign with respect to the untwisted sector ones, due to the insertion of the matrix $\mathcal{I}$. The duals to states with bosonic oscillators are the minus-sign duplicates of the ones in (5.3), (5.4). The fermionic oscillators are instead half-integer modded, so in the identification there are different phases with respect to the untwisted sector. The first excited string states ($r = 1/2$) correspond to the R-R forms and are made with two fermions acting on the vacuum. In field theory we have, for generic half-integer r:

$$\eta_r^{1,a}\eta_r^{2,b}|0,p^+\rangle_T \longleftrightarrow \frac{1}{\sqrt{2JN^{J+2}}}\left(\sum_{l=0}^{J}\mathrm{Tr}\left[\chi^a\tilde{Z}^l\psi^b Z^{J-l}\right]e^{\frac{2\pi i r l}{J}} - \sum_{l=0}^{J}\mathrm{Tr}\left[\psi^a Z^l\chi^b\tilde{Z}^{J-l}\right]e^{\frac{2\pi i r l}{J}}\right). \tag{5.9}$$

In the energy formula (3.23) for these states we have two separate contributions from the zero point energy (the energy of the tachyonic ground state) and the square root. The zero point energy gives no perturbative contribution, as we have seen. The square root can be easily reproduced at first order in field theory, the calculation being exactly the same as that in the untwisted sector. The only difference is the half-integer phase factor. For example, for the first excited state $r = 1/2$, we have a factor of $1/2$ in the argument of the exponentials, whose square gives the $1/4$ difference in the first nontrivial term in the square root.

Finally, we notice that, for operators with bosonic insertions, the relative sign in (5.4) discriminates between untwisted and twisted sector. For operators with fermionic insertions, due to the difference in modding of the phase factors, we can write two more operators by reversing signs in (5.6) and (5.9). These operators must have anomalous dimensions that diverge for large 't Hooft coupling, since there are no string states corresponding to them. Notice that an operator is almost-BPS only if all contributions in λ to the anomalous dimension cancel, even those that are usually neglected in the dilute-gas approximation because suppressed by powers of J. In our case, already at first order in λ, there are $\lambda/J \sim \sqrt{\lambda}$ contributions to the anomalous dimensions that are canceled only with a specific choice of signs. To check this, it is convenient to write the generic operator with two fermionic impurity in $\mathcal{N} = \triangle$ notations as:

$$O_{k,q} = \frac{1}{\sqrt{2JN^{J+2}}}\sum_{l=0}^{J}\mathrm{Tr}\left[\chi^1 Z^l\chi^2 Z^{J-l}\mathcal{I}^k\right]e^{\frac{2\pi i q l}{J}}\,, \tag{5.10}$$

where χ^i and Z are 2N$\times$ 2N matrices, $k = 0, 1$. $\mathcal{I}$ commutes with all Z's and anticommutes with χ^i. Most of the leading contribution in $\langle O_{k,q}\bar{O}_{k,q}\rangle$ come from contracting the l term in the sum with itself. However, in the BPS case $q = k = 0$, a λ/J contribution, coming from the exchange graphs for the two impurities, cancels when summing the $l = 0/l = 0$, $l = J/l = J$ and $l = 0/l = J$ contractions. For generic q and k, the $l = 0/l = J$

contraction carries an extra $(-1)^k e^{2\pi i q}$ factor, where the $(-1)^k$ is due to the fact that $\mathcal{I}$ anticommutes with the impurities. We thus obtain an exact cancellation of the λ/J terms only for $(-1)^k e^{2\pi i q} = 1$. This correlates the integer or half-integer modding of the phase with the insertion of $\mathcal{I}$.

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A. Scalar fields stability bound in PP-wave

In this appendix we reproduce the result for the mass of the tachyon in the gravity approximation.[6]

The "massive" scalar lagrangian density in the background (1.1) reads ($\sqrt{-g} = 1$):

$$\mathcal{L} = -\frac{1}{2}\left[2\partial_+\phi\partial_-\phi + f^2x^2\partial_-\phi\partial_-\phi + \partial_I\phi\partial_I\phi + m^2\phi^2\right]. \tag{A.1}$$

When $f = 0$ we recover the flat space lagrangian in the front-form of dynamics, in which x^+ plays the role of time variable.

We define the energy as the generator of x^+ translations. It is evident that $\xi_+ = \xi_+^\mu\partial_\mu = \partial_+$ is a Killing vector for the background (1.1), so that the following quantity (T being the stress tensor for gravity plus scalar fluctuations):

$$H = \int dx^- d^8x T^{+\mu} g_{\mu\nu}\xi_+^\nu = \int dx^- d^8x T_{+-} \tag{A.2}$$

is (formally) conserved in x^+. The scalar field contribution in T_{+-} corresponds to the canonical hamiltonian density derived from (A.1):

$$\mathcal{H} = \pi\partial_+\phi - \mathcal{L} = \frac{1}{2}\left[f^2x^2\partial_-\phi\partial_-\phi + \partial_I\phi\partial_I\phi + m^2\phi^2\right]. \tag{A.3}$$

Let us stress that, as noted in [16], this does not uniquely fix the energy functional. In fact we can define an "improved" stress tensor (which is also conserved), whose relevant component reads:

$$T_{+-}^{im} = T_{+-} + \beta(\Box - \partial_+\partial_-)\phi^2. \tag{A.4}$$

Here, referring to the notations in [16], we have used the fact that $D_+\partial_-\phi = \partial_+\partial_-\phi$, that $R_{+-} = 0$, and that, being $V(\phi) \approx m^2\phi^2$, in our case the critical point is $\phi_0 = 0$ and so $V(\phi_0) = 0$.

[6] Here and in the following we will use the fact that the non-trivial components of the connection and curvature are ($g^{--} = \mathrm{f}^2x_I^2$): $\Gamma^m_{+I} = -\mathrm{f}^2x^I\delta^m_-$, $\Gamma^m_{++} = \mathrm{f}^2x^I\delta^m_I$, $R_{I++J} = -\mathrm{f}^2\delta_{IJ}$, $R_{++} = 8\mathrm{f}^2$. In particular the scalar curvature is zero in the background.

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A.1 Equation of motion and its solution

The equation of motion following from (A.1) is:

$$(\Box - m^2)\phi \equiv \left[2\partial_+\partial_- + f^2x^2\partial_-\partial_- + \partial_I\partial_I - m^2\right]\phi = 0\,. \tag{A.5}$$

This is of first order in ∂_+, and it is well now (see for example [17]) that the associated Cauchy problem can be solved by imposing mixed initial-boundary conditions. Here we are interested in those solutions which go to zero when $x^I \to \infty$ (these are required if we want our energy functionals to be actually convergent and so effectively conserved in x^+). Then we choose (see also [18]):

$$\phi \approx e^{-ip_+x^+}e^{ip_-x^-}e^{-cx^2}\,, \tag{A.6}$$

which is an acceptable solution[7] of (A.5) provided that:

$$c = \frac{f|p_-|}{2}\,; \qquad 2p_+p_- - 16c - m^2 = 0\,. \tag{A.7}$$

Thus the general form of the selected kind of solution reads:

$$\phi(x^+, x^-, x^I) = \int_{-\infty}^{\infty} dp\theta(p)\left[a(p)e^{-\frac{i}{2p}(m^2+8fp)x^+}e^{ipx^-} + a^*(p)e^{\frac{i}{2p}(m^2+8fp)x^+}e^{-ipx^-}\right]e^{-\frac{fp}{2}x^2}\,. \tag{A.8}$$

Here we have relabelled $p_- = p$ and imposed the reality condition on ϕ.

From the relations (A.7) we can see that in the gravity limit $\alpha' p^+ f \to 0$ the vacuum state in the twisted sector of our type-0B model (whose light cone energy is $p_+ \approx 4f - (1/\alpha' p^+)$) can be viewed as a scalar fluctuation of mass $m^2 = -(2/\alpha')$.

A.2 The energy functional

Let us now evaluate the hamiltonian on the solution (A.8). After integration in x^- we find:

$$H = \int d^8x \int dp\theta(p)a(p)a^*(p)[2f^2p^2x^2 + m^2]e^{-fpx^2}\,. \tag{A.9}$$

The integration over x^I is easily performed in spherical coordinates using the fact that (for $a > 0$):

$$\int_0^{\infty} dy y^n e^{-ay} = \frac{n!}{a^{(n+1)}}\,. \tag{A.10}$$

We see that:

$$H = 3\Omega\int_0^{\infty}\frac{dp}{(fp)^4}\left[8pf + m^2\right]a(p)a^*(p)\,. \tag{A.11}$$

($\Omega > 0$ results from the integration over the transverse angular variables).

Each mode of momentum $p_-(= p^+)$ gives a positive contribution to the energy, provided that:

$$m^2 > -8(fp^+)\,. \tag{A.12}$$

[7] Let us outline that the very general normalizable solution contains a product of Hermite polynomials depending on the transverse coordinates. The choice (A.6) corresponds to the lowest of such polynomials. We are interested in the string theory vacuum, without any insertion of bosonic zero-mode oscillators.

In particular we see that the state corresponding to the twisted sector vacuum has, in the gravity limit, a mass which violates this bound, giving a negative light-cone energy.

Let us now turn to the "improved" energy functional:

$$H_{im} = H + \beta \int dx^- d^8x (\Box - \partial_+ \partial_-) \phi^2 \,. \tag{A.13}$$

By using the equation of motion we find that:

$$H_{im} = (4\beta + 1)H + 2\beta \int dx^- d^8x (\partial_+ \phi \partial_- \phi - \phi \partial_+ \partial_- \phi) \,. \tag{A.14}$$

On the solution (A.8) we have:

$$H_{im} = H. \tag{A.15}$$

We see that the improved energy does not give any other bound on the mass parameter.

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